



ISE 220 – Engineering Economics

Ch. 3 – Understanding Money Management

Effective versus Nominal Rates



University of Economics

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2013, İzmir



Agenda

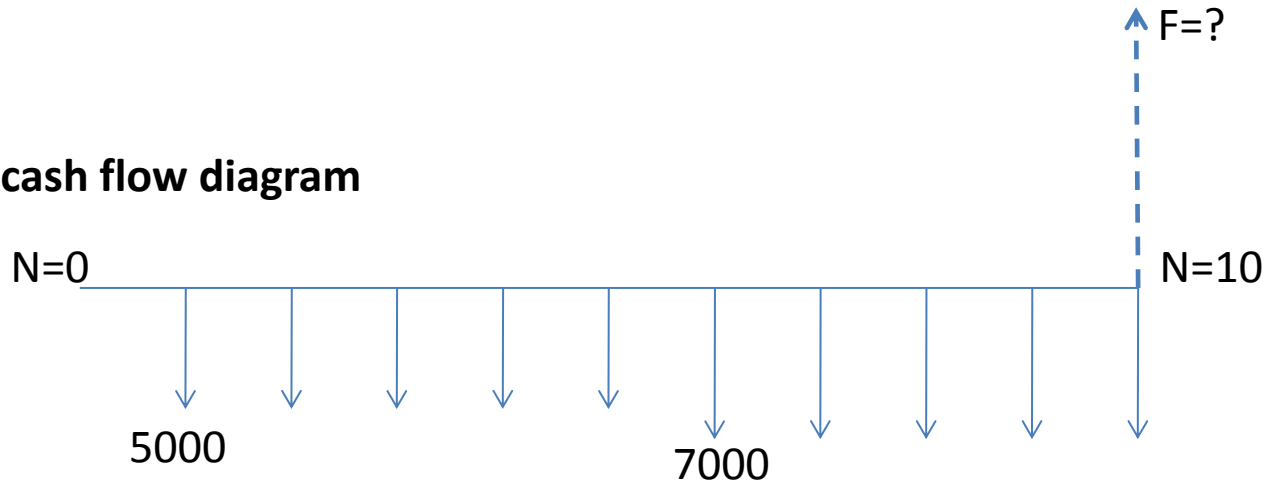
- How to solve Midterm questions
- Understanding Money Management & Market Interest Rates
 - Nominal versus Effective Interest Rate
 - Compounded per Interest rates
- Class work



Market Interest Rates

Suppose that you are working in US and deposit in a bank which says have interest rate of 12% / year. If you deposit \$5000 each year for the first five years, and \$7000 for the second five years, how much money will you have at the end of ten years?

1. Draw cash flow diagram



2. Dissect into notation

$$F = 5000(F/A, 12\%, 10) + 2000(F/A, 12\%, 5)$$



Market Interest Rates

Suppose that you are working in US and deposit in a bank which says have **interest rate of 12% / year**. If you deposit \$5000 each year for the first five years, and \$7000 for the second five years, how much money will you have at the end of ten years?

If compounding not indicated be cautious:

1. Nominal interest rate is 12% (compounded how ?)
2. Annual percentage rate is 12% (compounded how ?)
3. Annual Interest Rate (compounded how ?)
4. Annual **effective** yield (Yes it is given)

ASK or Calculate What is compounding !!!

EFFECTIVE INTEREST RATE will be used for the period.

1. Effective annual interest rate is 12% (Compounded yearly)



Rules

When Solving a problem

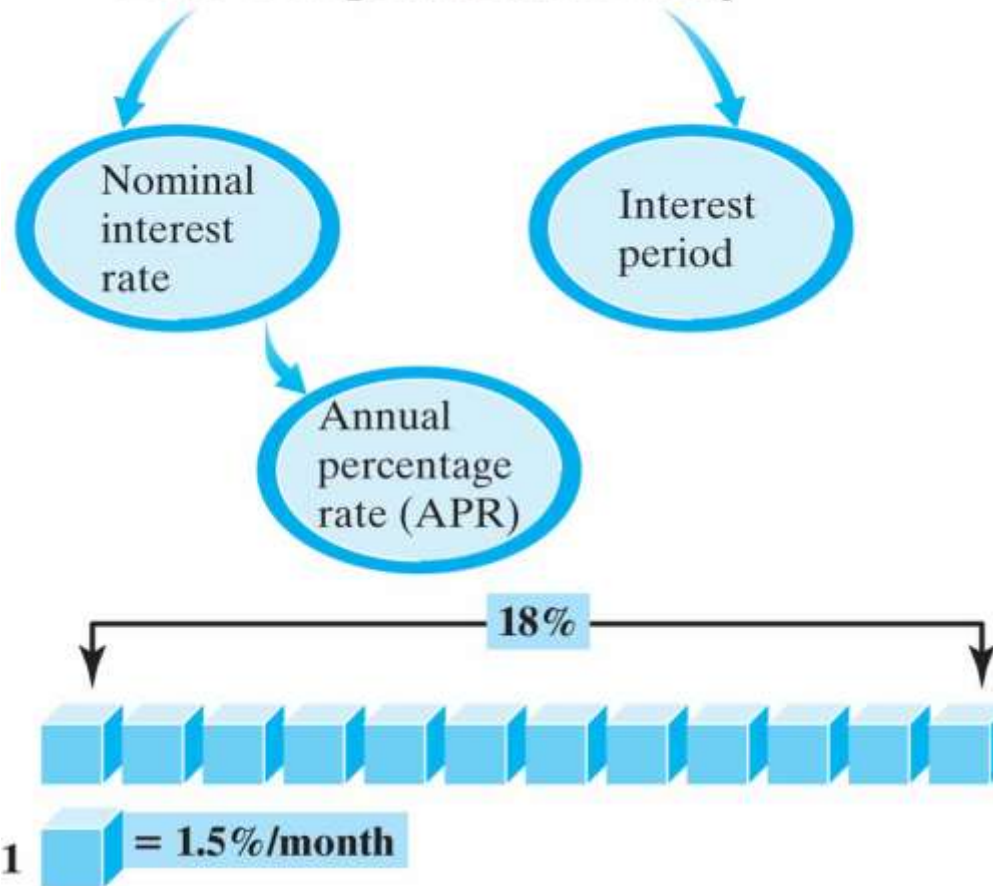
Interest Rate should be effective interest rate for the payment period.

If it is given differently – Equalize it to the period



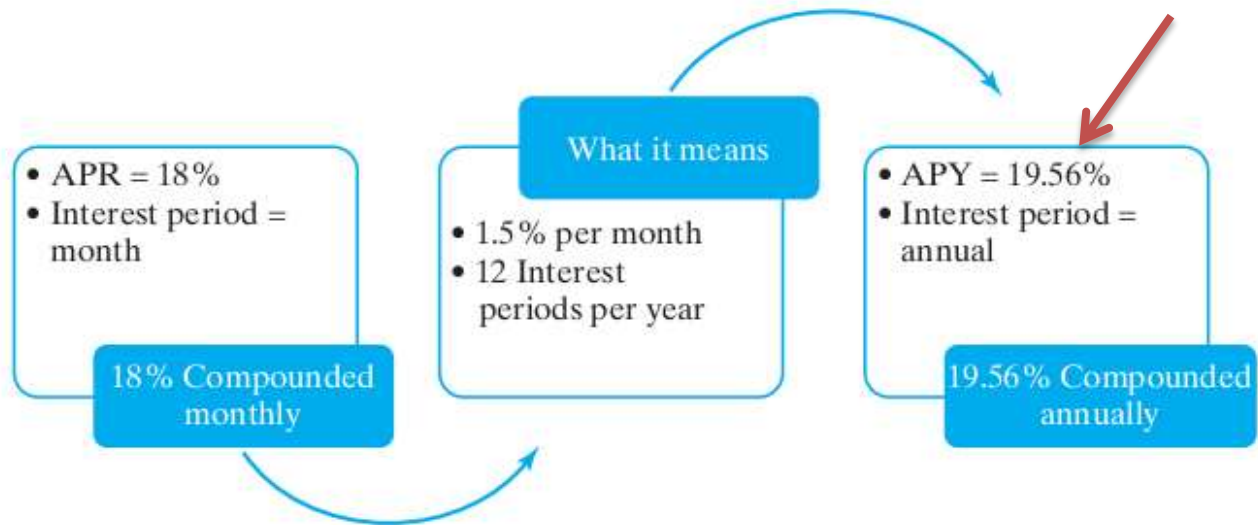
Annual Effective Yield = ?

18% Compounded Monthly





Annual Effective Yield





Nominal versus Effective yield

TABLE 3.1 Annual Effective Yields at Various Compounding Intervals

Nominal Rate	Compounding Frequency				
	Annually	Semiannually	Quarterly	Monthly	Daily
4%	4.00%	4.04%	4.06%	4.07%	4.08%
5%	5.00%	5.06%	5.09%	5.12%	5.13%
6%	6.00%	6.09%	6.14%	6.17%	6.18%
7%	7.00%	7.12%	7.19%	7.23%	7.25%
8%	8.00%	8.16%	8.24%	8.30%	8.33%
9%	9.00%	9.20%	9.31%	9.38%	9.42%
10%	10.00%	10.25%	10.38%	10.47%	10.52%
11%	11.00%	11.30%	11.46%	11.57%	11.63%
12%	12.00%	12.36%	12.55%	12.68%	12.75%



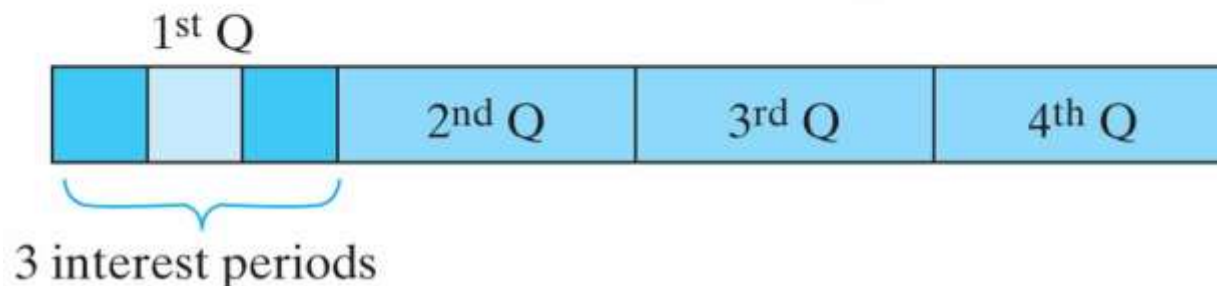
Effective Int. Rate / Quarter

Yearly nominal is 8%

Case 1: 8% compounded monthly

Payment Period = Quarterly

Interest Period = Monthly



Given: $r = 8\%$;

$K = 4$ quarterly payments per year;

$C = 3$ interest periods per quarter;

$M = 12$ interest periods per year.

$$\begin{aligned}i &= [1 + r/CK]^C - 1 \\ &= [1 + 0.08/(3)(4)]^3 - 1 \\ &= 2.013\% \text{ per quarter.}\end{aligned}$$



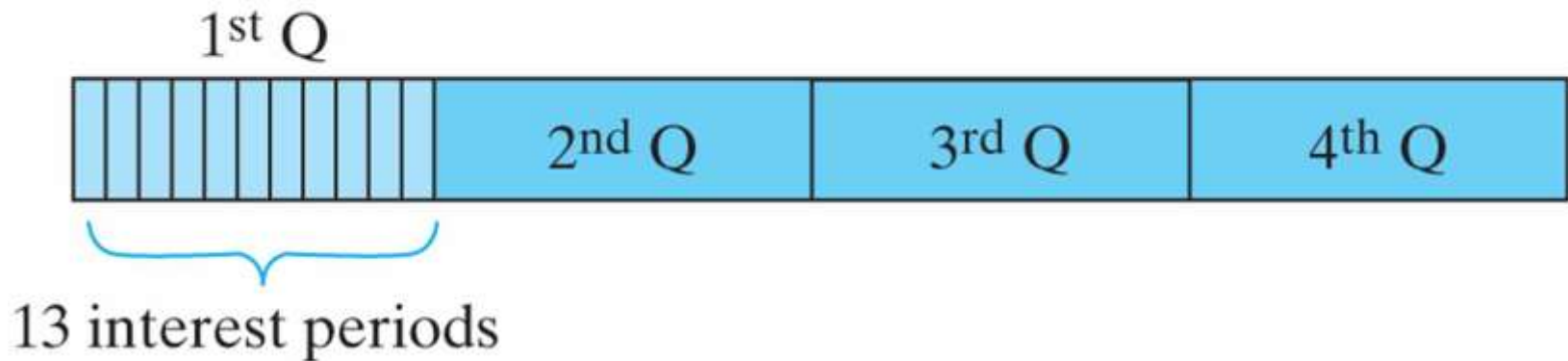
Effective Int. Rate / Quarter

Yearly nominal is 8%

Case 2: 8% compounded weekly

Payment Period = Quarterly

Interest Period = Weekly





Effective Int. Rate / Month

EXAMPLE 3.4 Calculating Auto Loan Payments

Suppose you want to buy a car. You have surveyed the dealers' newspaper advertisements, and the one in Figure 3.5 has caught your attention. You can afford to make a down payment of \$2,678.95, so the net amount to be financed is \$20,000.

- What would the monthly payment be?
- After the 25th payment, you want to pay off the remaining loan in a lump-sum amount. What is this lump sum?

- 8.5% Annual Percentage Rate!
48 month financing on all Mustangs in stock. 60 to choose from.
- ALL READY FOR DELIVERY!
Prices starting as low as \$21,599.
- You just add sales tax and 1% for dealer's freight. We will pay the tag, title, and license.
- Add 4% sales tax = \$863.96
- Add 1% dealer's freight = \$215.99
- Total purchase price = **\$22,678.95**



Dealers.com

Resolve Assuming Annual Effective Interest Rate is 8.5%
In order to learn REVERSE COMPOUNDING



Reverse Compounding

$$(1+i)^{12} = 1 + 0.085$$

$$i = (1 + 0.085)^{1/12} - 1$$

$$i = 1.0068 - 1 = 0.0068 = 0.68\%$$

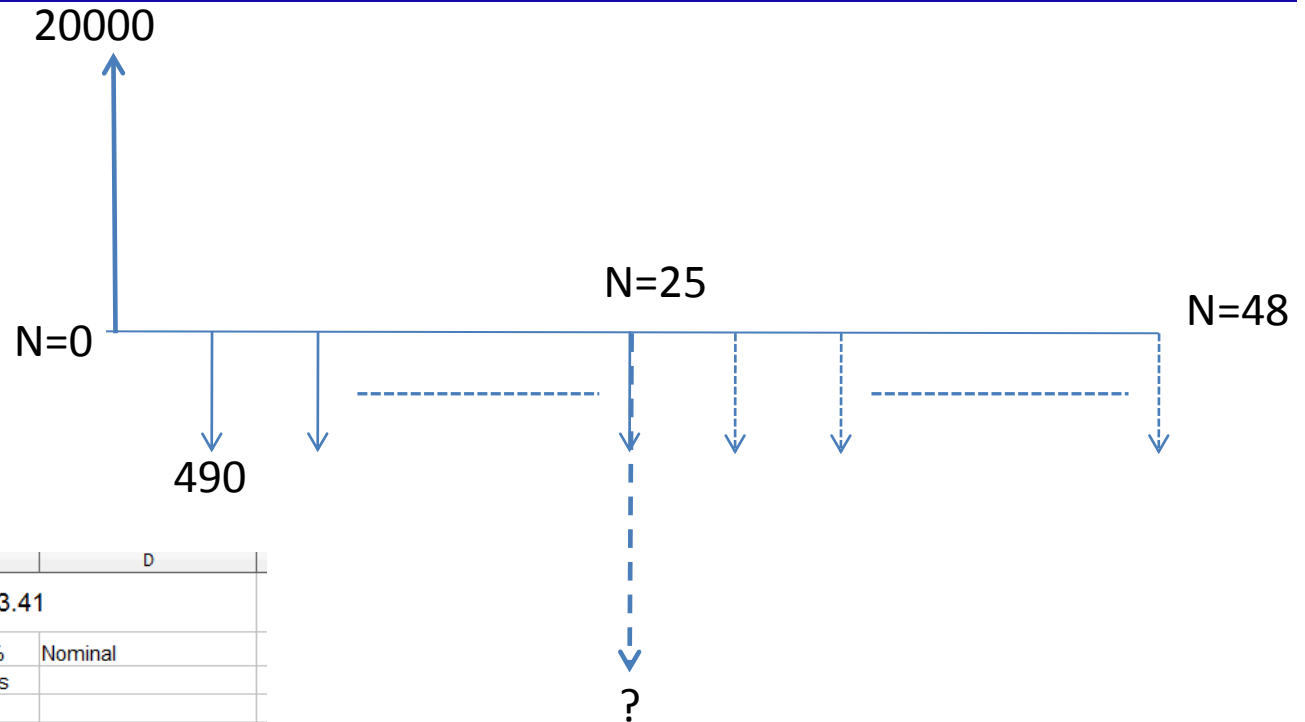
	A	B	C	D
1	Ford Finance 3.4			
2	$i =$	0.085	8.50%	Nominal
3	$N =$	48	Months	
4	$P =$	20000	\$	
5				
6	$i_m =$	0.006821	0.68%	$(1+i)^{1/12} - 1$
7	$A =$	-490.00TL		$20000(A/P, 0.68\%, 48)$
8				
9	Period (n)	Cash Flow	Interest	Balance
10	0	20,000.00TL		20,000.00TL
11	1	-490.00TL	136.43TL	19,646.43TL
12	2	-490.00TL	134.02TL	19,290.44TL
13	3	-490.00TL	131.59TL	18,932.02TL
14	4	-490.00TL	129.14TL	18,571.16TL
15	5	-490.00TL	126.68TL	18,207.84TL
16	6	-490.00TL	124.20TL	17,842.04TL



$$A = 20000(A/P, 0.68\%, 48) = 490$$



Early payment



	A	B	C	D
1	Ford Finance 3.41			
2	$i =$	0.085	8.50%	Nominal
3	$N =$	48	Months	
4	$P =$	20000	\$	
5				
6	$i_m =$	0.006821	0.68%	$(1+i)^{1/12} - 1$
7	$A =$	-490.00TL		$20000(A/P, 0.68\%, 48)$
8	Early Pay	10,397.76TL		$490(P/A, 0.68\%, 23)$
9				
10	Period (n)	Cash Flow	Interest	Balance
11	0	20,000.00TL		20,000.00TL
12	1	-490.00TL	136.43TL	19,646.43TL
13	2	-490.00TL	134.02TL	19,290.44TL
14	3	-490.00TL	131.59TL	18,932.02TL
15	4	-490.00TL	129.14TL	18,571.16TL
16	5	-490.00TL	126.68TL	18,207.84TL
17	6	-490.00TL	124.20TL	17,842.04TL

$$P = 490(P/A, 0.68\%, 23)$$



Compounding

- Discrete compounding

r – Given yearly interest rate as compounded differently then yearly.

M – Compounding periods per year

C – Number of compounding periods per payment period

i – Interest rate per payment period

$$i = (1 + r/M)^C - 1$$

- Continuous (infinite) compounding (Every second)

$$i = e^{r/K} - 1$$

Banks may like more frequent compounding



Remember Rule

When Solving a problem

Interest Rate should be effective interest rate for the payment period.



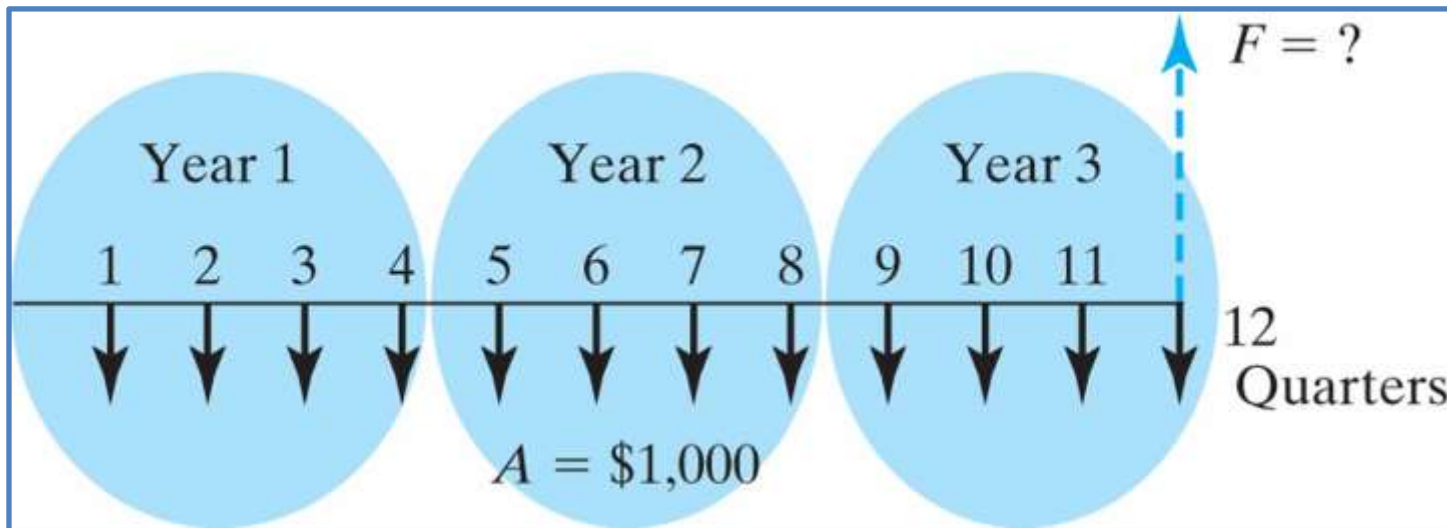
Compounding

EXAMPLE 3.5 Compounding Occurs More Frequently than Payments Are Made

Suppose you make equal quarterly deposits of \$1,000 into a fund that pays interest at a rate of 12% compounded monthly. Find the balance at the end of year 3.

$$i = (1 + 0.01)^3 - 1 \\ = 3.03\%$$

$$F = 1000(F/A, i, 12) \\ = 14,216.24$$





Debt Management

$P = 5000$ (Loan amount)

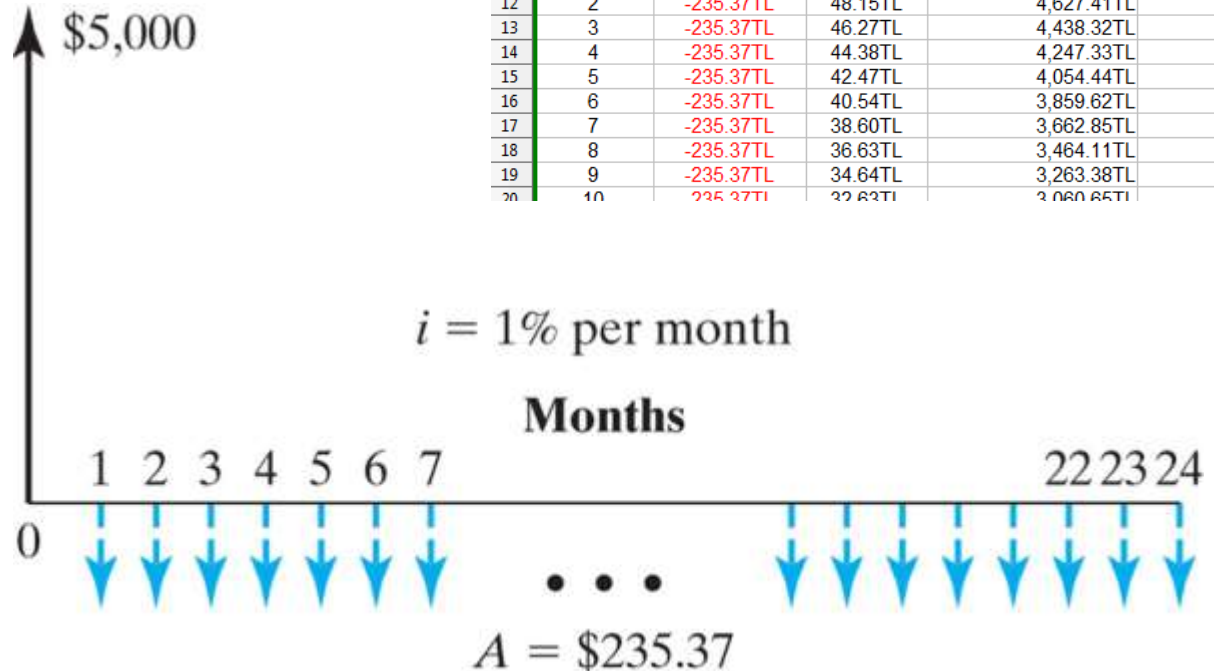
$N = 24$ months (Contract period)

APR = 12%

Monthly Installments = ?

	A	B	C	D	E
1	See the cash flow 3.7				
2	$i =$	0.12	12.00%	Nominal	
3	$N =$	24	Months		
4	$P =$	5000	\$		
5	$i_{m1} =$	0.01	1.00%	Compounded monthly	Which one
6	$i_{m2} =$	0.009489	0.95%	Compounded yearly	
7	$A =$	-235.37TL		20000(A/P, 0.68%, 48)	
8					
9	Period (n)	Cash Flow	Interest	Balance	
10	0	5,000.00TL		5,000.00TL	
11	1	-235.37TL	50.00TL	4,814.63TL	
12	2	-235.37TL	48.15TL	4,627.41TL	
13	3	-235.37TL	46.27TL	4,438.32TL	
14	4	-235.37TL	44.38TL	4,247.33TL	
15	5	-235.37TL	42.47TL	4,054.44TL	
16	6	-235.37TL	40.54TL	3,859.62TL	
17	7	-235.37TL	38.60TL	3,662.85TL	
18	8	-235.37TL	36.63TL	3,464.11TL	
19	9	-235.37TL	34.64TL	3,263.38TL	
20	10	-235.37TL	32.63TL	3,060.65TL	

Check in spreadsheet to see how interest rate is compounded





Debt Management

P = 5000 (Loan amount)

N = 24 months (Contract period)

APR = 12%

Monthly Installments = 253.37

Construct the loan payment schedule by showing the remaining balance, interest payment, and principal payment at the end of each period over the life of the loan.

More spreadsheet formulas

	A	B	C	D	E	F	G
1		Example 3.8 Loan Repayment Schedule					
2							
3							
4		Contract amount	\$5,000.00		Total payment		\$5,648.82
5		Contract period	24		Total interest		\$648.82
6		APR (%)	12				
7		Monthly Payment	(\$235.37)				
8							
9							
10			Payment No.	Payment Size	Principal Payment	Interest payment	Loan Balance
11			1	(\$235.37)	(\$185.37)	(\$50.00)	\$4,814.63
12			2	(\$235.37)	(\$187.22)	(\$48.15)	\$4,627.41
13			3	(\$235.37)	(\$189.09)	(\$46.27)	\$4,438.32
14			4	(\$235.37)	(\$190.98)	(\$44.38)	\$4,247.33
15			5	(\$235.37)	(\$192.89)	(\$42.47)	\$4,054.44
16			6	(\$235.37)	(\$194.82)	(\$40.54)	\$3,859.62
17			7	(\$235.37)	(\$196.77)	(\$38.60)	\$3,662.85
18			8	(\$235.37)	(\$198.74)	(\$36.63)	\$3,464.11
19			9	(\$235.37)	(\$200.73)	(\$34.64)	\$3,263.38
20			10	(\$235.37)	(\$202.73)	(\$32.63)	\$3,060.65
21			11	(\$235.37)	(\$204.76)	(\$30.61)	\$2,855.89
22			12	(\$235.37)	(\$206.81)	(\$28.56)	\$2,649.08
23			13	(\$235.37)	(\$208.88)	(\$26.49)	\$2,440.20
24			14	(\$235.37)	(\$210.97)	(\$24.40)	\$2,229.24
25			15	(\$235.37)	(\$213.08)	(\$22.29)	\$2,016.16
26			16	(\$235.37)	(\$215.21)	(\$20.16)	\$1,800.96
27			17	(\$235.37)	(\$217.36)	(\$18.01)	\$1,583.60
28			18	(\$235.37)	(\$219.53)	(\$15.84)	\$1,364.07
29			19	(\$235.37)	(\$221.73)	(\$13.64)	\$1,142.34
30			20	(\$235.37)	(\$223.94)	(\$11.42)	\$918.40
31			21	(\$235.37)	(\$226.18)	(\$9.18)	\$692.21
32			22	(\$235.37)	(\$228.45)	(\$6.92)	\$463.77
33			23	(\$235.37)	(\$230.73)	(\$4.64)	\$233.04
34			24	(\$235.37)	(\$233.04)	(\$2.33)	\$0.00
35							
36							
37							
38							
39							
40							
41							
42							



Debt Management



EXAMPLE 3.9 Buying a Car: Paying in Cash versus Taking a Loan

Consider the following two options proposed by an auto dealer:

- **Option A:** Purchase the vehicle at the normal price of \$26,200 and pay for the vehicle over 36 months with equal monthly payments at 1.9% APR financing.
- **Option B:** Purchase the vehicle at a discounted price of \$24,048 to be paid immediately. The funds that would be used to purchase the vehicle are presently earning 5% annual interest compounded monthly.

Which option is more economically sound?



Debt Management

List Price = 26200

APR = 1.9% (Comp. monthly)

$$A = 26200(A/P, (1.9\%/12), 36) \\ = 749.29$$

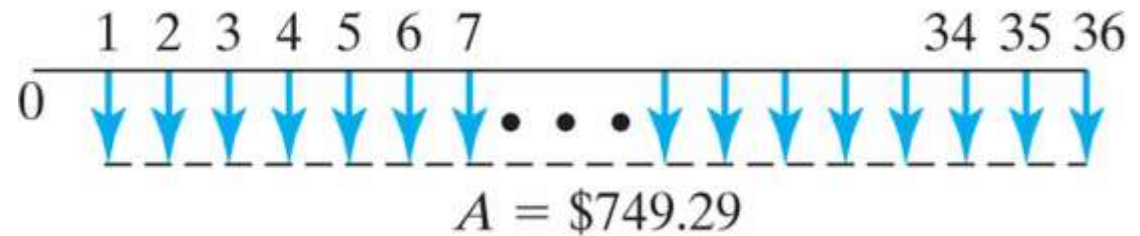
Alternating 5% effective monthly

$$P = 749.29(P/A, (5\%/12), 36) \\ = 25000$$

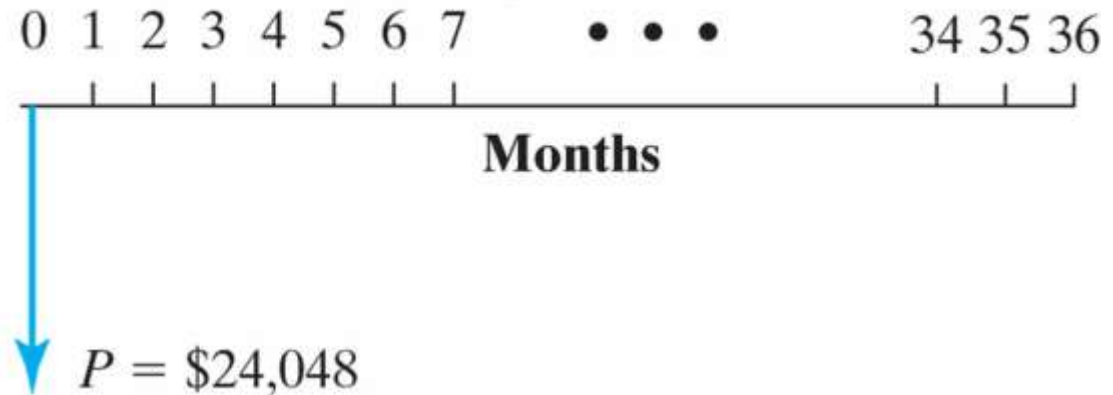
Discounted Cash Price = 24048

	A	B	C	D
1	Buying a car - Cash versus Loan - 3.8			
2	$i =$	0.001583	0.16%	1.9% / 12
3	$A =$	-749.29TL	\$	26200(A/P, 1.9%/12, 36)
4				
5	$i_A =$	0.004167	0.42%	5% / 12
6	$P_A =$	25,000.66TL	\$	(P/A, 5%/12, 36)
7	$P_B =$	24,048.00TL		Cash discounted price
8				
9	Period (n)	Cash Flow	Interest	Balance
10	0	25,000.66TL		25,000.66TL
11	1	-749.29TL	104.17TL	24,355.54TL
12	2	-749.29TL	101.48TL	23,707.73TL
13	3	-749.29TL	98.78TL	23,057.22TL
14	4	-749.29TL	96.07TL	22,404.00TL
15	5	-749.29TL	93.35TL	21,748.06TL
16	6	-749.29TL	90.62TL	21,089.90TL

Option A



Option B





Debt Management

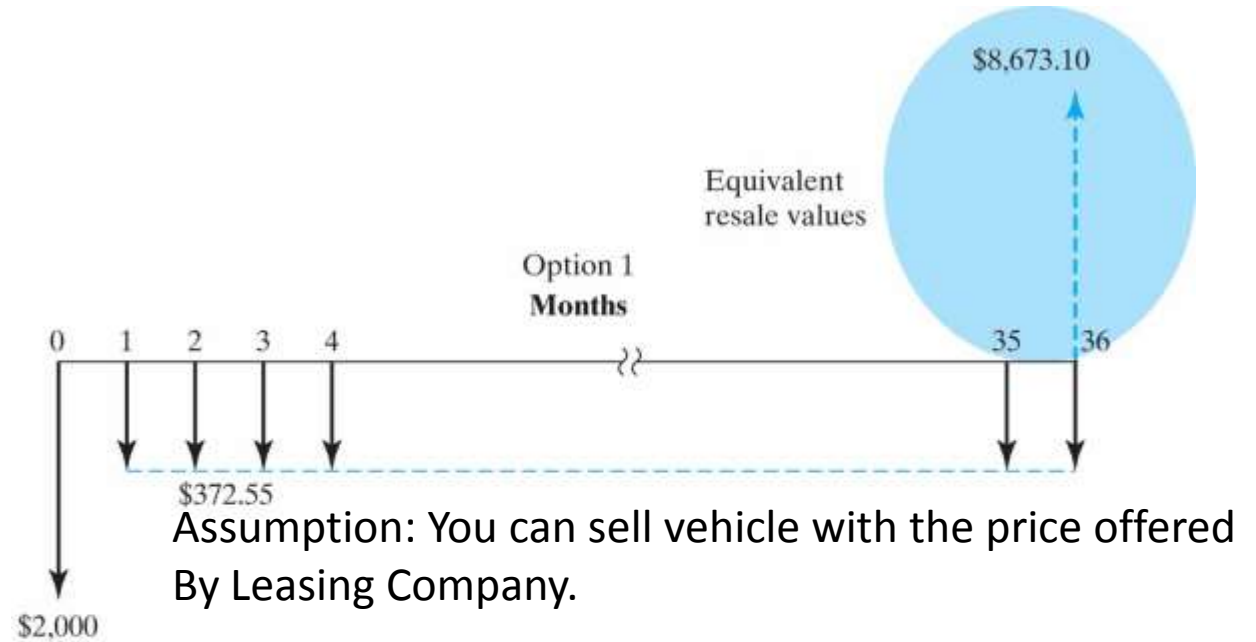
EXAMPLE 3.10 Buying versus Leasing a Car

Buying versus Leasing		
	Option 1 Debt Financing	Option 2 Lease Financing
Price	\$14,695	\$14,695
Down payment	\$2,000	\$0
APR (%)	3.6%	
Monthly payment	\$372.55	\$236.45
Length	36 months	36 months
Fees		\$495
Cash due at lease end		\$300
Purchase option at lease end		\$8,673.10
Cash due at signing	\$2,000	\$731.45

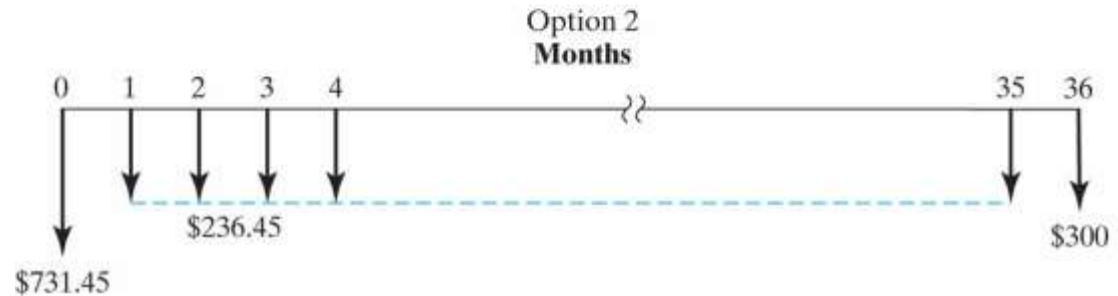


Debt Management

Crediting



Leasing





Debt Management

Market Interest Rate= 6% compounded monthly

- Debt Financing:

$$\begin{aligned}P_{\text{debt}} &= \$2,000 + \$372.55(P/A, 0.5\%, 36) \\ &\quad - \$8,673.10(P/F, 0.5\%, 36) \\ &= \$6,998.47\end{aligned}$$

- Lease Financing:

$$\begin{aligned}P_{\text{lease}} &= \$495 + \$236.45 + \$236.45(P/A, 0.5\%, 35) \\ &\quad + \$300(P/F, 0.5\%, 36) \\ &= \$8,556.90\end{aligned}$$



Class work

- 3.42 Suppose that \$1,500 is placed in a bank account at the end of each quarter over the next 20 years. What is the account's future worth at the end of 20 years when the interest rate is 8% compounded
- (a) semiannually?
 - (b) monthly?
 - (c) continuously?
- (d) Suppose you wish to pay a lump sum at the end of 15 years. To have the same FV at the end of year 20 how much should you pay if interest rate is given as 8% compounded quarterly?



Questions

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